MATH2050C Assignment 7

Deadline: March 12, 2018.

Hand in: 3.7 no. 3c, 10, 15, 16.

Section 3.7 no. 3ac, 7, 10, 11, 12, 15, 16.

Supplementary Problems

1. An infinite series $\sum_n x_n$ is called **absolutely convergent** if $\sum_n |x_n|$ is convergent. Show that an absolutely convergent infinite series is convergent but the convergence of $\sum_n x_n$ does not necessarily imply the convergence of $\sum_n |x_n|$.

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Basic Examples of Infinite Series

Let $\sum_{n=1}^{\infty} x_n$ be an infinite series. Its n-th partial sum s_n is given by $\sum_{k=1}^{n} x_k$. An infinite series $\sum_{n=1}^{\infty} x_n$ is called **convergent/divergent** if the sequence $\{s_n\}$ is convergent/divergent. When an infinite series converges, we use $\sum_{n=1}^{\infty} x_n$ to denote the limit $\lim_{n\to\infty} s_n$. Thus, the notation $\sum_{n=1}^{\infty} x_n$ has two meanings; first it is the notation for an infinite series, and second, it is the ultimate sum of the infinite series (provided it converges).

Sometimes, $\sum_{n=1}^{\infty} x_n$ is replaced by the simpler $\sum_{n}^{\infty} x_n$ or $\sum_n x_n$.

Example 1

$$\sum_{n=1}^{\infty} (-1)^{n+1}$$

is divergent.

Example 2 For $\alpha \in (0, 1)$,

$$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha} \ .$$

Example 3

$$\sum_{n=1}^{\infty} \frac{1}{n^t}$$

is convergent if and only if $t > 1$.

Example 4 The alternating harmonic series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$$

is convergent.

You should know the proofs behind these examples.

Comparison Theorem Let $0 \leq x_n \leq y_n$ for all n. Then (a) $\sum_{n=1}^{\infty} y_n$ converges implies $\sum_{n=1}^{\infty} x_n$ converges; and (b) $\sum_{n=1}^{\infty} x_n$ diverges implies $\sum_{n=1}^{\infty} y_n$ diverges.